Understanding Number Sense

General: The main aims of this lecture are:

1) Introduce students to the ubiquitous nature of number in their environment
2) Introduce students to the distinction between approximate and exact number
3) Introduce students to the idea that the approximate number sense is present in infants and animals
4) Show students how approximate number sense is related to math abilities

Slides 2-7: Encountering number in our environment

Activity: *Ask students where they see number in each picture. Make sure that they mention as many different ways as possible.*

Slide 2: Students could name the number of flowers but also the number of petals on each flower or the total number of petals. The latter could be exactly counted or approximated.

Slide 3: It is clear that it is quite complicated to count the number of circles and that approximation is much easier in some cases.

Slide 4: Students could count the number of shoes, number of pairs of shoes, number of different kinds of shoes etc. This slides nicely illustrates that there are numerous ways in which the same objects can be grouped and enumerated.

Slide 5: Students could list various things that can be counted exactly or approximated in these image.

Slide 6: Numbers are used for labeling people in a team.

Slide 7: Numbers are used to express prices.

Slide 8: Summarize the distinction between exact number and approximate number

Emphasize that we usually think about exact number when we talk about numbers and math; however, the approximate number system is much simpler because it does not require us to have language and know how to count.

Slide 9: How to test exact and approximate number

All the students are probably familiar with (standardized) math tests that examine age-appropriate math abilities. The approximate number sense can be tested using Panamath. If the students have not done Panamath, you can explain what the test is about (people see images with blue and yellow dots flash briefly and they have to say whether there were more blue or more yellow dots).

Slide 10: Typical features of our approximate number sense

Activity: Ask the students to explain the graph. What is shown on the x-axis? What is the ratio? The left display of Panamath is a difficult comparison of a ratio of 1.05 (17 yellow dots and 18 blue dots). The right display of Panamath is an easy comparison of a ratio of 2 (24 blue dots and 12 yellow dots). What is shown on the y-axis? Students should say that the number of correct answers for each ratio increases as the ratio becomes larger, i.e. as the comparison becomes easier.

Slide 11-12: Why do we have an approximate number sense?

Activity: Have students come up with reasons why we may have an approximate number sense.

Slide 11: A few examples for possible scenarios from our everyday life.

Slide 12: Emphasize the ecological and evolutionary reason for having an approximate number sense. It is critical for animals' and cavemen's survival to be able to tell which food patch has more food. It is also essential to compare the size of a group of predators to the size of one's own group to determine if it's better to flee or to fight. Finally, it is critical to track the number of one's offspring to ensure survival of one's own children.

Slide 13: Animals' approximate number sense

Activity: Ask the students to explain the graph. What is shown on the x-axis? What is shown on the y-axis? What are the different curves? Monkeys are overall a little less accurate on the Panamath task than humans, but their performance still gets better as the ratio between the numbers gets larger, i.e. the task becomes easier. This suggests that they use the same approximate number sense as humans even though they don't speak and cannot count or do formal math.

Slide 14-16: Infants' approximate number sense

Activity: How could we test infants' approximate number sense? What are the difficulties? How could we overcome them?

Slide 14: Students could say that infants cannot talk and cannot understand task instructions. Possible ways to overcome the problems would be to have them look at things and measure their interest in different numbers of objects, hide a number of toys in a box and see how many times they reach in to retrieve the toys, have them crawl or walk to different buckets with different numbers of food items (they should prefer the bucket with more food).

Slide 15-16: Infants are able to tell the difference between the numbers of elements in two sets of objects if the difference is large enough. For example, if 6-month-olds are repeatedly shown different images with 8 dots their looking times to these images decreases. They become bored with the repetition of "8", they

habituate. However, if they are then shown a new image of 16 dots they look longer to that than a new image of 8 dots. This shows that even 6-month-olds can already tell the difference between 8 and 16 without being able to count at all. Importantly, this also works if infants are first shown different images with 16 dots, so it is not just a preference to look to more dots.

Emphasize that the examples of monkeys and infants show that the approximate number sense does not rely on language and that it is present even without knowing anything about counting and formal math.

Slide 17-19: The relationship between the approximate number sense and formal math

There is a strong association between the approximate number sense and formal math abilities throughout the life span. The three slides show different age groups at which these relationships have been found. In each case, the better people's approximate number sense (i.e., the lower their w!), the better their math abilities.

Slide 17: College-aged adults: SAT-Quantitative scores correlate with precision of the approximate number sense, but there is no correlation between SAT-Verbal and precision of the approximate number sense.

Slide 18: Elementary school children: Standardized math scores correlate with precision of the approximate number sense. In the paper, the researchers show that this relationship also holds when we take differences in children's IQ and other cognitive skills that develop at this age into account.

Slide 19: Preschool children: Even before children enter school and start learning math formally, there is a relationship between children's approximate number sense and their early math abilities such as counting and doing simple arithmetic.

Slide 20: Why is there a relationship between the approximate number sense and formal math?

We do not know exactly why the approximate number sense and math abilities are related, but we can speculate about the reasons.

1) Our approximate number sense may provide us with a sense of ordinal direction, i.e. numbers are ordered and adding objects makes numbers larger and taking away objects makes them smaller.
2) Our approximate number sense may also help us in estimating the answers to mathematical problems.
3) Our approximate number sense may provide us with a sense of confidence in our numerical abilities. If we are good at estimating numbers, we may be more confident in our answers to math questions in general.
4) Our approximate number sense may encourage us to engage in more math-related activities. If we think that we are good at estimating numbers, we may also think that we are good at other types of activities involving numbers.

Slide 21: Future research and open questions

This slide should give the students the sense that research is exciting and that there are many more things that we do not know about our approximate number sense and math abilities. These are just a few examples of open questions at the time of making these slides. Our hope is that they will be answered in the near future.

Activity: Have the students come up with their own questions and ideas for future research studies. This can be expanded into a longer assignment in which students may do a literature search to see if anybody has done any research on their question.

References:

Cantlon, J. F., & Brannon, E. M. (2006). Shared system for ordering small and large numbers in monkeys and humans. *Psychological Science, 17*(5), 401-406.

Halberda, J., Mazzocco, M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature, 455*(7213), 665-668.

Libertus, M. E., Feigenson, L., Halberda, J. (2011). Preschool acuity of the approximate number system correlates with school math ability. *Developmental Science,* DOI: 10.1111/j.1467-7687.2011.01080.x

Libertus, M. E., Odic, D., Halberda, J. (in prep). Intuitive Sense of Number Correlates With Math Scores on College-Entrance Examination.

Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition, 74*(1), B1-B11.

Further resources:

1) A brief review paper about current research on infants' approximate number sense:

> Libertus, M. E., & Brannon, E. M. (2009). Behavioral and Neural Basis of Number Sense in Infancy. *Current Directions in Psychological Science, 18*(6), 346-351.

2) A review paper about current research on number and the brain. Summarizes work with humans and monkeys:

> Nieder, A., & Dehaene, S. (2009). Representation of number in the brain. *Annu Rev Neurosci, 32*, 185-208.

3) Groundbreaking studies that showed that even indigenous people in the Amazon have an approximate number sense but cannot due exact math:

> Gordon, P. (2004). Numerical Cognition Without Words: Evidence from Amazonia. *Science, 306*(5695), 496-499.
>
> Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science, 306*(5695), 499-503.